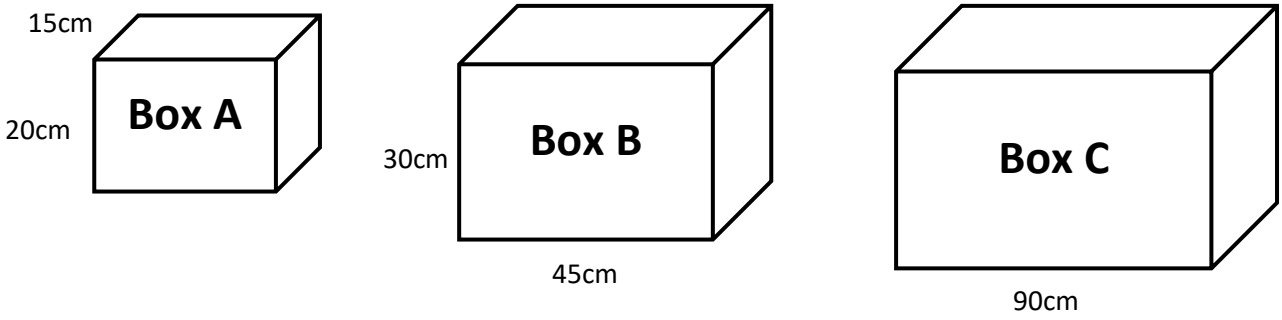
Elf Box Packing Problem

1. The Elves have decided to streamline their packaging and have decided to use three SIMILAR boxes for most of the toys. *Diagram not to scale.*



(a) Work out the scale factor between each pair of boxes:

To get from Box A to Box B, you multiply by ***1.5*** (★)

To get from Box B to Box C, you multiply by ***2*** (★)

To get from Box A to Box C, you multiply by ***3*** (★)

To get from Box C to Box B, you multiply by ***1/2*** (★★)

To get from Box C to Box A, you multiply by ***1/3*** (★★)

To get from Box B to Box A, you multiply by ***2/3*** (★★★)

(Hint: For tricky problems think about fractions)

(b) Work out the lengths of the missing sides of the boxes

Box	Width (cm)	Height (cm)	Depth (cm)
A	***30***	20	15
B	45	30	***22.5***
C	90	***60***	***45***

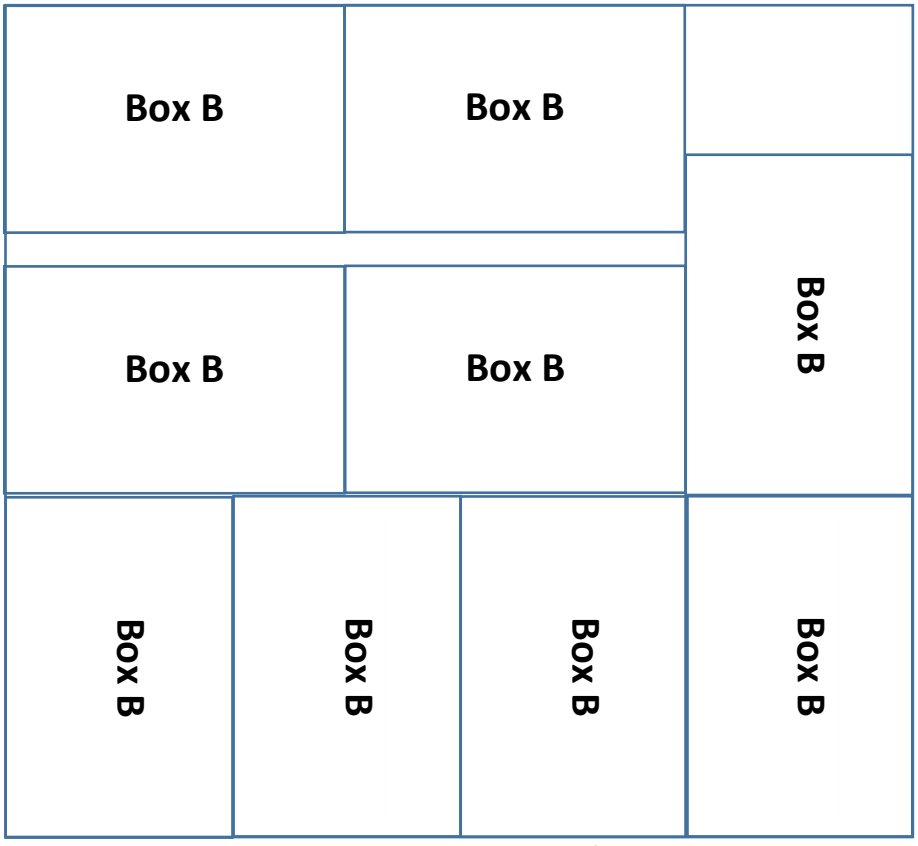
2. The toy boxes are stored in packing crates for ease of transportation. Each box is laid on its back so that they can check the contents quickly. The packing crate has a base which is 1.2m by 1.1m.

(a) What is the area covered by Box A *(Hint: $A = H \times W$)*? ***600*** cm^2

(b) What is the area of the base of the packing crate in cm^2 ? ***13200*** cm^2

(c) Divide your result from (b) by your result for (a). This is how many toy boxes can be laid out in the bottom of the packing crate. Draw a diagram to show how you could arrange them. ***22 boxes, there is more than one possible solution***

(d) Use an accurate diagram to convince Eddie that Ellie is correct.



3. Occasionally a present needs a different box.

(a) A new type of tricycle needs a box which has a length of 1 metre, a height of $\frac{4}{5}$ m and a depth of $\frac{1}{3}$ metre. What is the volume of this box in m^3 ? Show all your working out
(*Hint: V = H × W × D)*

$$1 \times \frac{4}{5} \times \frac{1}{3} = \frac{4}{15}$$

(b) What is the volume of 15 of these boxes? 4

(c) A triangular prism has a cross-section of $\frac{4}{9}$ m^2 and a length of $\frac{3}{8}$ m. What is the volume of the prism? Give your answer in its simplest form.

$$\frac{4}{9} \times \frac{3}{8} = \frac{12}{72} = \frac{1}{6}$$

(d) A cylinder has a base of $\frac{3}{4}$ m^2 and a volume of $\frac{5}{12}$ m^3. What is the height of the cylinder?

$$\frac{5}{12} \div \frac{3}{4} = \frac{5}{12} \times \frac{4}{3} = \frac{20}{36} = \frac{5}{9}$$

4. There are different teams assigned to toy packing, depending on the urgency.

(a) The Everyday Elf team can pack $2\frac{1}{5}$ tonnes of toys in one hour. How much can they pack in $\frac{1}{4}$ of an hour? $2\frac{1}{5} \times \frac{1}{4} = \frac{11}{5} \times \frac{1}{4} = \frac{11}{20}$

(b) The Elite Elf squad pack at a rate which is $1\frac{1}{3}$ times faster than the Everyday Elves. How many tonnes do they pack in one hour?

$$2\frac{1}{5} \times 1\frac{1}{3} = \frac{11}{5} \times \frac{4}{3} = \frac{44}{15} = 2\frac{14}{15}$$

(c) How long will it take each team to pack 15 tonnes of toys?

Everyday Elves: $15 \div \frac{11}{5} = 15 \times \frac{5}{11} = \frac{75}{11} = 6\frac{9}{11}$

Elite Elves: $15 \div \frac{44}{15} = 15 \times \frac{15}{44} = \frac{225}{44} = 5\frac{5}{44}$

5. Challenge

The Elves need to fill a section of the sleigh with a mixture of Boxes A, B & C. The section has a height of 2m, a width of 3m and a depth of 1.8m.

How can you fill this with:

(a) Just Box A?
(b) Just Box B?
(c) Just Box C?
(d) A mixture of boxes to minimise wasted space?

These were designed as open discussion pieces. There is more than one solution in most cases.

If students are struggling, they should be directed back to Question 2 and the 2D version. This should help them think of strategies.